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Localization of Gravity on Dilatonic Domain Walls: Addendum to "Solitons in Brane Worlds"

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Abstract

We supplement the discussion on localization of gravity on dilatonic domain walls in "Solitons in Brane Worlds" (Nucl. Phys. **B576**, 106, hep-th/9911218) by giving unified and coherent discussion which combines the result of this paper and the expanded results in the author's subsequent papers in an attempt to avoid misleading readers. We also discuss the possible string theory embeddings of the Randall-Sundrum type brane world scenarios through non-dilatonic and dilatonic domain walls, which straightforwardly follows from the author's previous works but was not elaborated explicitly.

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In Ref. [1], we argued that dilatonic domain walls with $\Delta < -2$ (see just below Eq. (4) for definition of Δ) localize gravity, since the potential term in the Schrödinger-type equation satisfied by the metric fluctuation is volcano-shaped just like that of the Randall-Sundrum (RS) domain wall [2, 3]. In the subsequent paper [4], we found out that gravity can be localized around the positive tension domain wall with $\Delta < 0$, since the effective gravitational constant in one lower dimensions is nonzero for such case. Then, later it is realized [5] that the graviton can be localized on the dilatonic domain wall for any values of Δ as long as the tension of the domain wall is positive, since for such case the normalizable Kaluza-Klein (KK) zero mode exists and the effective gravitational constant in one lower dimensions is nonzero. We feel that these different (but expanded) results scattered around our previous works may cause confusion among readers. Furthermore, perhaps to worsen the confusion, in Ref. [6], which later reproduces these our previous papers, it is argued that gravity can be trapped on the dilatonic domain walls only for the $\Delta \leq -2$ case, since only for such case normalizable graviton KK mode exists. It is a purpose of this brief note to gather the results on the localization of gravity on the dilatonic domain wall, which are scattered around our different previous papers, to give unified and coherent discussion in order to prevent misleading readers, and hopefully to clarify the contradicting result of Ref. [6]. We also elaborate on the possible embeddings of the RS type brane world scenario into string theories through dilatonic and non-dilatonic domain walls, which follows straightforwardly from our previous works but was not elaborated explicitly.

The action for the dilatonic domain wall with an arbitrary dilaton coupling parameter a in arbitrary spacetime dimensions D is

$$S_{\rm bulk} = \frac{1}{2\kappa_D^2}\int d^D x\sqrt{-G}\left[\mathcal{R}_G - \frac{4}{D-2}\partial_M\phi\partial^M\phi + e^{-2a\phi}\Lambda\right]. \tag{1}$$

The worldvolume action for the dilatonic domain wall with energy density (or tension) σ_{DW} is

$$S_{\rm DW} = -\int d^{D-1}x\sqrt{-\gamma}\sigma_{DW}e^{-a\phi}, \tag{2}$$

where γ is the determinant of the induced metric $\gamma_{\mu\nu} = \partial_\mu X^M\partial_\nu X^N G_{MN}$ on the domain wall worldvolume.

The dilatonic domain wall solution in the standard warp factor form is [1, 5]:

$$\begin{aligned} G_{MN}dx^M dx^N &= \mathcal{W}\left[-dt^2 + dx_1^2 + \cdots + dx_{D-2}^2\right] + dy^2, \\ \phi = \frac{1}{a}\ln(1+Ky), &\qquad \mathcal{W} = (1+Ky)^{\frac{8}{(D-2)^2a^2}}, \end{aligned} \tag{3}$$

where the constant K can take following positive or negative value independently on both sides of the domain wall:

$$K = \pm\frac{(D-2)a^2}{2}\sqrt{\frac{D-2}{4(D-1)-a^2(D-2)^2}\Lambda} = \pm\frac{(D-2)a^2}{2}\sqrt{-\frac{\Lambda}{2\Delta}}, \tag{4}$$

where $\Delta \equiv (D-2)a^2/2 - 2(D-1)/(D-2)$. From this expression for K, one can see that the domain solution of the form (3) exists only for $\Delta < 0$ [$\Delta > 0$] when $\Lambda > 0$ [$\Lambda < 0$]. The boundary condition at $y = 0$ fixes the domain wall tension σ_{DW} to be the following fine-tuned form [5]:

$$\sigma_{DW} = \frac{1}{\kappa_D^2} \frac{4}{(D-2)a^2}(K_- - K_+), \tag{5}$$

where K_- [K_+] denotes the value of K at $y < 0$ [$y > 0$]. So, σ_{DW} is positive [negative] when $K_- > 0$ and $K_+ < 0$ [$K_- < 0$ and $K_+ > 0$], in which case there are naked singularities at finite nonzero y [no singularity at $y \neq 0$]. And σ_{DW} is zero when K_+ and K_- have the same sign. The $(D-1)$-dimensional gravitational constant has non-zero value given by $\kappa_{D-1}^2 = \frac{\Delta+4}{2}\sqrt{-\frac{\Lambda}{2\Delta}}\kappa_D^2$ when $\sigma_{DW} > 0$; otherwise, $\kappa_{D-1}^2 = 0$ [4, 5]. So, a necessary condition for localizing gravity on the domain wall is $\sigma_{DW} > 0$, which is possible for *any values of* Δ.

By redefining the transverse coordinate of the domain wall, one can put the solution into the following conformally flat form [1, 5]:

$$\begin{aligned} &G_{MN}dx^M dx^N = \mathcal{C}\left[-dt^2 + dx_1^2 + \cdots + dx_{D-2}^2 + dz^2\right], \\ &\phi = \frac{(D-2)a}{2(\Delta+2)}\ln(1+\bar{K}z), \qquad \mathcal{C} = (1+\bar{K}z)^{\frac{4}{(D-2)(\Delta+2)}}, \end{aligned} \tag{6}$$

where the constant $\bar{K}$ is given by

$$\bar{K} = \eta\frac{(D-2)^2a^2 - 4}{(D-2)^2a^2}K = \pm\eta(\Delta+2)\sqrt{-\frac{\Lambda}{2\Delta}}, \tag{7}$$

where $\pm$ is the same as $\pm$ in Eq. (4) and the sign ambiguity $\eta = \pm$ resulting from the coordinate transformation to the conformally flat metric can be fixed to be $\eta = +$ by demanding that κ_{D-1}^2 should be calculated to take the same values as above. From now on, we consider only the $\mathbf{Z}_2$-symmetric solution with $\sigma_{DW} > 0$, for which $\kappa_{D-1} \neq 0$. So, in the remainder of this paper, we let $\mathcal{W} = (1+K|y|)^{\frac{8}{(D-2)^2a^2}}$ and $\mathcal{C} = (1+\bar{K}|z|)^{\frac{4}{(D-2)(\Delta+2)}}$ with K and $\bar{K}$ respectively given by Eqs. (4) and (7) with $-$ sign. [For the choice of $+$ sign, $\sigma_{DW} < 0$ and $\kappa_{D-1} = 0$.]

One can determine whether gravity can be localized on the domain wall also by studying the linearized Einstein equations satisfied by the metric fluctuation $h_{\mu\nu}(x^\rho, z) = \hat{h}^{(m)}_{\mu\nu}(x^\rho)\mathcal{C}^{-(D-2)/4}\psi_m(z)$ in the RS gauge. When $\eta^{\mu\nu}\partial_\mu\partial_\nu\hat{h}^{(m)}_{\mu\nu} = m^2\hat{h}^{(m)}_{\mu\nu}$, $\psi_m(z)$ satisfies the following Schrödinger-type equation [1, 5]:

$$-\frac{d^2\psi_m}{dz^2} + V(z)\psi_m = m^2\psi_m, \tag{8}$$

with the potential given by

$$V(z) = \frac{D-2}{16}\left[(D-6)\left(\frac{\mathcal{C}'}{\mathcal{C}}\right)^2 + 4\frac{\mathcal{C}''}{\mathcal{C}}\right] = -\frac{(1+\Delta)\Lambda}{2\Delta(1+\bar{K}|z|)^2} + \frac{2\bar{K}}{\Delta+2}\delta(z). \tag{9}$$

Note, the potential expression in Eq. (24) of Ref. [1] is related to this expression through $Q = \frac{\Delta}{\Delta+2}\bar{K} = \pm\Delta\sqrt{-\frac{\Lambda}{2\Delta}}$ with the sign $\pm$ being that in Eq. (7). [In Refs. [1, 7, 4], we considered $Q > 0$ case, only, namely $-$ sign choice ($\sigma_{DW} > 0$) for $\Delta < 0$ and $+$ sign choice ($\sigma_{DW} < 0$) for $\Delta > 0$. In this paper, we consider more general case, as we did in Ref. [5].] Note, $V(z)$ has attractive [repulsive] δ-function potential term when $\sigma_{DW} > 0$ [$\sigma_{DW} < 0$] for *any value of* Δ. So, for *any value of* Δ, the graviton KK zero mode $\psi_0 \sim \mathcal{C}^{(D-2)/4}$, satisfying the boundary condition $\psi_0'(0^+) - \psi_0'(0^-) = \frac{2\bar{K}}{\Delta+2}\psi_0(0)$, is normalizable (i.e., $\int dz|\psi_0|^2 < \infty$) [2], as long as we choose $-$ sign in Eq. (7) [5]. Even for the $\sigma_{DW} < 0$ case, the graviton KK zero mode is still given by $\psi_0 \sim \mathcal{C}^{(D-2)/4}$ for any values of Δ but is not normalizable. Particularly, for the $\Delta < -2$ case with $\sigma_{DW} > 0$, $V(z)$ resembles the volcano potential of the RS model [2, 3] and therefore has the same KK mode spectrum structure and the same type of correction to the Newtonian gravitational potential from the massive KK modes as the RS domain wall case, as was noted in Ref. [1]. However, note that even for the $\Delta > -2$ case with $\sigma_{DW} > 0$, the normalizable KK zero mode exists, even if $V(z)$ does not take the volcano potential form. The structure of the massive KK mode spectrum for the $\Delta > -2$ case is discussed in Ref. [1]. [Note, the possible tachyonic modes for the $\Delta > -1$ case mentioned in Ref. [1] do not exist, since the result of Ref. [8] ensures that $m^2 \geq 0$. For the $\Delta > 0$ case, Ref. [1] considers the case of repulsive δ-function potential (i.e., $+$ sign choice in Eq. (7)), but the structure of the massive KK mode spectrum discussed in Ref. [1] still remains the same even for the attractive δ-function potential case.] However, in the $\Delta > 0$ case (i.e., the $\Lambda < 0$ case), the $(D-1)$-dimensional effective action has diverging cosmological constant term [5]. It is found out [5] that to avoid such divergence one has to cut off the transverse space by introducing additional domain wall with the fine-tuned tension between $z = 0$ and the curvature singularity.

To sum up, the normalizable graviton KK zero mode exists and the effective gravitational constant in one lower dimension is nonzero for *any values of* Δ when $\sigma_{DW} > 0$, but the introduction of additional domain wall is required for the $\Delta > 0$ case to remove diverging cosmological constant term in the effective action in one lower dimensions. And when $\sigma_{DW} \leq 0$, the graviton KK zero mode is not normalizable and the gravitational constant in one lower dimension is zero for *any values of* Δ. [Note, the graviton KK modes for the $\Delta = -2$ case was not studied by us, but was later studied in Ref.

[2]Note, for the positive tension domain walls, $\bar{K}$ is positive [negative] for $\Delta < -2$ [$\Delta > -2$] and therefore the metric (6) is well-defined over the interval $-\infty < z < \infty$ [$\bar{K}^{-1} \leq z \leq -\bar{K}^{-1}$], over which the normalization integral is integrated

[6]. It is shown there that the normalizable graviton KK zero mode exists and there is the continuum of the massive KK mode with mass gap for the $\Delta = -2$ case with $\sigma_{DW} > 0$.]

We now comment on the results of Ref. [6] which contradict the results of our previous papers. The authors of Ref. [6] claim that the normalizable graviton KK zero mode exists only for the $\Delta \leq -2$ case with $\sigma_{DW} > 0$, for which $\bar{K}$ (which corresponds to k in Ref. [6]) is positive, and the introduction of additional domain wall is necessary for the $\Delta > -2$ case in order to trap gravity. However, our careful analysis shows that gravity can be trapped even for the $\Delta > -2$ case, for which $k < 0$ for the positive tension domain wall, and the introduction of additional domain wall is necessary for the $\Delta > 0$ case not to trap gravity but to remove diverging cosmological constant term in the $(D-1)$-dimensional effective action. As necessary conditions for trapping gravity, Ref. [6] states that the conformal factor $\mathcal{C}$ should vanish at large $|z|$ and the δ-function source should have a positive tension so that the potential term in the Schrödinger equation can be volcano-like. However, Ref. [6] fails to notice that $\mathcal{C}$ vanishes at finite $|z|$ (instead of at $|z| = \infty$) for the $\Delta > -2$ and $\sigma_{DW} > 0$ case. And as we discussed, the potential term needs not be volcano-like in order to support the normalizable graviton KK zero mode.

We now discuss string theory embeddings of the above-discussed domain wall solutions which localize gravity. For this purpose, we begin by studying the uplifting of such domain walls to the dilatonic p-branes in $D' > D$ dimensions. The action for the dilatonic p-brane is given by

$$S_p = \frac{1}{2\kappa^2_{D'}} \int d^{D'}x \sqrt{-\hat{G}} \left[\mathcal{R}_{\hat{G}} - \frac{4}{D'-2} (\partial\varphi)^2 - \frac{1}{2 \cdot (p+2)!} e^{2a_p\varphi} F^2_{p+2} \right]. \tag{10}$$

The dilatonic p-brane solution is characterized by the parameter $\Delta_p = (D'-2)a_p^2/2 + 2(p+1)(D'-p-3)/(D'-2)$. (See Eq. (2) of Ref. [1] for the explicit solution with this convention for the action.) Any single-charged branes in string theories are special cases of the dilatonic p-branes. The dilatonic p-branes can also be realized from any intersecting branes (with N numbers of constituents) in string theories by setting the charges of the constituent branes equal to one another and then compactifying along the relative transverse directions (and possibly overall transverse and longitudinal directions). When the dilatonic p-branes are embedded in string theories in such a manner, the parameter Δ_p takes only the special values $4/N$ and is invariant under the compactification on a Ricci flat manifold involving consistent truncation [9]. So, the dilatonic domain wall solutions obtained from (intersecting) branes in string theories through the Scherk-Schwarz dimensional reduction [10] on a Ricci flat manifold have $\Delta = 4/N$, only. However, one can have dilatonic domain wall solutions in string theories with different values of Δ through the compactification on spheres. By

compactifying the D'-dimensional dilatonic p-brane (in the near horizon limit) with $\Delta_p = 4/N$ on $S^{D'-p-2}$, one obtains the dilatonic domain wall in $(p+2)$-dimensions with the dilaton coupling parameter a and Δ given by [1]

$$
\begin{aligned}
|a| &= \frac{2}{p}\sqrt{\frac{2(D'-2)-N(p+1)(D'-p-3)}{2(D'-p-2)-N(D'-p-3)}}, \\
\Delta &= -\frac{4(D'-p-3)}{2(D'-p-2)-N(D'-p-3)}.
\end{aligned}
\tag{11}
$$

By applying these results, we now elaborate on various possible string theory embeddings of dilatonic domain walls in five dimensions.

First, we discuss the case of the dilatonic domain walls with $\Delta = 4/N$ ($N \in \mathbf{Z}^+$). To obtain such domain walls, we (i) start with (intersecting) branes in ten or eleven dimensions (with equal constituent brane charges) with at least 3-dimensional (overall) longitudinal space and at least 1-dimensional (overall) transverse space, and then (ii) compactify the extra (overall) longitudinal directions (if any) and all the relative transverse directions (if any), and perform the Scherk-Schwarz dimensional reduction along the extra (overall) transverse directions (if any). The possible cases are as follows:

- $\Delta = 4$ case: M5-brane; NS5-brane; Dp-branes with $3 \leq p \leq 8$.
- $\Delta = 2$ case: $(3|\mathrm{M5},\mathrm{M5})$; $(3|\mathrm{NS5},\mathrm{NS5})$; $(p-1|\mathrm{NS5},\mathrm{D}p)$ with $p = 4,5,6$; $(p-2|\mathrm{D}p,\mathrm{D}p)$ with $p = 5,6$; $(p-1|\mathrm{D}p,\mathrm{D}(p+2))$ with $p = 4,5$; $(p|\mathrm{D}p,\mathrm{D}(p+4))$ with $p = 3,4$.
- $\Delta = 4/3$ case: M5$\perp$M5$\perp$M5 with 3-dimensional overall longitudinal space; D4$\perp$NS5$\perp$NS5; D5$\perp$D5$\perp$NS5; NS5$\perp$NS5$\perp$D5.

It is interesting to note that the spacetime metric for the 5-dimensional domain wall solution studied in Ref. [11] is the $\Delta = 4/3$ case of the dilatonic domain wall metric. So, the property of the graviton KK modes in such domain wall bulk background is the $\Delta = 4/3$ case of what we have studied.

Second, we discuss the string theory embeddings of the 5-dimensional dilatonic domain walls with $\Delta \neq 4/N$. To obtain such domain walls, we (i) start with (intersecting) branes in ten or eleven dimensions (with equal constituent brane charges) with at least 3-dimensional (overall) longitudinal space and at least 3-dimensional (overall) transverse space, (ii) compactify the relative transverse directions (if any), the extra longitudinal directions and possibly some of overall transverse directions to obtain the dilatonic 3-brane in $D' \geq 7$ dimensions, and then (iii) compactify on $S^{D'-5}$. The values of the dilaton coupling parameter a and Δ of the resulting 5-dimensional dilatonic

domain wall are the $p = 3$ case of Eq. (11):

$$
\begin{aligned}
|a| &= \frac{2}{3}\sqrt{\frac{2(D'-2)-4N(D'-6)}{2(D'-5)-N(D'-6)}}, \\
\Delta &= -\frac{4(D'-6)}{2(D'-5)-N(D'-6)}.
\end{aligned}
\tag{12}
$$

A particularly interesting case is the non-dilatonic domain wall ($a = 0$ case) of the RS model [2, 3]. It is straightforward to check that a in Eq. (12) can be zero only for the $D' = 10$ and $N = 1$ case, i.e., the S^5-reduction of D3-brane. This possibility of embedding the RS model was later studied in Ref. [12]. The remaining cases of string theory embeddings of dilatonic domain walls with $\Delta \neq 4/N$ are as follows:

- $N = 1$ case:
  (1) starting with M5-brane, one compactifies two of the longitudinal directions and then compactifies the transverse space on $T^n \times S^{4-n}$ ($n = 0, 1, 2$) to obtain 5-dimensional domain wall with $\Delta = -\frac{4(3-n)}{5-n} = -\frac{12}{5}, -2, -\frac{4}{3}$;
  (2) starting with NS5-brane, one compactifies two of the longitudinal directions and then compactifies the transverse space on $T^n \times S^{3-n}$ ($n = 0, 1$) to obtain 5-dimensional domain wall with $\Delta = -\frac{4(2-n)}{4-n} = -2, -\frac{4}{3}$;
  (3) starting with Dp-brane with $3 \leq p \leq 6$, one compactifies $p-3$ of the longitudinal directions and then compactifies the transverse space on $T^n \times S^{8-p-n}$ ($n \leq 6-p$) to obtain 5-dimensional domain wall with $\Delta = -\frac{4(7-p-n)}{9-p-n} = -\frac{8}{3}, -\frac{12}{5}, -2, -\frac{4}{3}$.

- $N = 2$ case:
  (1) starting with $(3|\mathrm{M5}, \mathrm{M5})$, one compactifies the relative transverse directions and then compactifies the overall transverse space on S^2 to obtain 5-dimensional domain wall with $\Delta = -2$;
  (2) starting with $(p-1|\mathrm{NS5}, \mathrm{D}p)$ with $p = 4, 5$, one compactifies the relative transverse directions and $p-4$ of the overall transverse directions and then compactifies the overall transverse space on S^2 obtain 5-dimensional domain wall with $\Delta = -2$.

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